[1] $\tan x + \cot x = \sec x \csc x$

$$\hookrightarrow = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}$$

$$= \frac{1}{\cos x \sin x}$$

$$= \sec x \csc x$$

[2] $\tan^2 \beta + \cot^2 \beta = \sec^2 \beta \csc^2 \beta - 2$

$$\hookrightarrow = \sec^2\beta - 1 + \csc^2\beta - 1$$

$$= \frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta} - 2$$

$$= \frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta \sin^2\beta} - 2$$

$$= \frac{1}{\cos^2\beta \sin^2\beta} - 2 = \sec^2\beta \csc^2\beta - 2$$

[3] $\sin^4 A + \cos^2 A = \cos^4 A + \sin^2 A$

$$\hookrightarrow = (1 - \cos^2 A)^2 + \cos^2 A$$

$$= 1 - 2\cos^2 A + \cos^4 A + \cos^2 A$$

$$= 1 - \cos^2 A + \cos^4 A$$

$$= \sin^2 A + \cos^4 A$$

[4] $\sec(-t) - \cos(-t) - \csc(-t) + \sin(-t) + \sin t \tan(-t) = \cos t \cot t$

$$\hookrightarrow = \sec t - \cos t + \csc t - \sin t - \sin t \tan t$$

$$= \frac{1}{\cos t} - \cos t + \frac{1}{\sin t} - \sin t - \sin t \frac{\sin t}{\cos t}$$

$$= \frac{1 - \cos^2 t}{\cos t} + \frac{1 - \sin^2 t}{\sin t} - \frac{\sin^2 t}{\cos t}$$

$$= \cancel{\frac{\sin^2 t}{\cos t}} + \frac{\cos^2 t}{\sin t} - \cancel{\frac{\sin^2 t}{\cos t}} = \cos t \frac{\cos t}{\sin t} = \cos t \cot t$$

[5] $(\tan\lambda - \sec\lambda\csc\lambda)(\cot\lambda - \sec\lambda\csc\lambda) = 1$

$$\text{L.S.} = \left(\frac{\sin t}{\cos t} - \frac{1}{\cos t\sin t}\right)\left(\frac{\cos t}{\sin t} - \frac{1}{\cos t\sin t}\right)$$

$$= \frac{\sin^2 t - 1}{\cos t\sin t} \cdot \frac{\cos^2 t - 1}{\cos t\sin t}$$

$$= \frac{-\cancel{\cos^2} t}{\cancel{\cos} t\cancel{\sin} t} \cdot \frac{-\cancel{\sin^2} t}{\cancel{\cos} t\cancel{\sin} t}$$

$$= 1$$

[6] $\dfrac{\cos^4\theta - \sin^4\theta}{\cos^2\theta} = 1 - \tan^2\theta$

$$\text{L.S.} = \frac{\overset{1}{\cancel{(\cos^2\theta + \sin^2\theta)}}(\cos^2\theta - \sin^2\theta)}{\cos^2\theta}$$

$$= 1 - \tan^2\theta$$

[7] $\dfrac{\sin B + \tan B}{1 + \cos B} = \tan B$

$$\text{L.S.} = \frac{\sin B + \frac{\sin B}{\cos B}}{1 + \cos B}$$

$$= \frac{\sin B\cos B + \sin B}{\cos B} \cdot \frac{1}{1 + \cos B}$$

$$= \frac{\sin B\cancel{(\cos B + 1)}}{\cos B} \cdot \frac{1}{\cancel{1 + \cos B}}$$

$$= \tan B$$

[8] $\dfrac{\cot y-\tan y}{\cos y+\sin y}=\dfrac{\cos y-\sin y}{\cos y\sin y}$

$$LS = \frac{\dfrac{\cos y}{\sin y}-\dfrac{\sin y}{\cos y}}{\cos y+\sin y}$$

$$= \frac{\cos^2 y-\sin^2 y}{\sin y\cos y}\cdot\frac{1}{\cos y+\sin y}$$

$$= \frac{\cancel{(\cos y+\sin y)}(\cos y-\sin y)}{\sin y\cos y\,\cancel{(\cos y+\sin y)}} = \frac{\cos y-\sin y}{\sin y\cos y}$$

[9] $\dfrac{1-\sin\alpha}{\cos\alpha}=\dfrac{1}{\sec\alpha+\tan\alpha}$

$$LS = \frac{1}{\sec\alpha+\tan\alpha}\cdot\frac{\sec\alpha-\tan\alpha}{\sec\alpha-\tan\alpha}$$

$$= \frac{\sec\alpha-\tan\alpha}{\cancel{\sec^2\alpha-\tan^2\alpha}\ 1}$$

$$= \frac{1}{\cos\alpha}-\frac{\sin\alpha}{\cos\alpha}$$

$$= \frac{1-\sin\alpha}{\cos\alpha}$$

[10] $\dfrac{(\sec C-\tan C)^2+1}{\sec C\csc C-\tan C\csc C}=2\tan C$

$$LS = \frac{\sec^2 C-2\sec C\tan C+\tan^2 C+1}{\csc C(\sec C-\tan C)}$$

$$= \frac{\sec^2 C-2\sec\tan C+\sec^2 C}{\csc C(\sec C-\tan C)}$$

$$= \frac{2\sec^2 C-2\sec C\tan C}{\csc C(\sec C-\tan C)}$$

$$= \frac{2\sec C\,\cancel{(\sec C-\tan C)}}{\csc C\,\cancel{(\sec C-\tan C)}}$$

$$= \frac{2}{\cos C}\cdot\frac{\sin C}{1} = 2\tan C$$