

[1]

$$\tan x + \cot x = \sec x \csc x$$

$$\begin{aligned} \hookrightarrow &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \\ &= \frac{1}{\cos x \sin x} \\ &= \sec x \csc x \end{aligned}$$

[2]

$$\tan^2 \beta + \cot^2 \beta = \sec^2 \beta \csc^2 \beta - 2$$

$$\begin{aligned} \hookrightarrow &= \sec^2 \beta - 1 + \csc^2 \beta - 1 \\ &= \frac{1}{\cos^2 \beta} + \frac{1}{\sin^2 \beta} - 2 \\ &= \frac{\sin^2 \beta + \cos^2 \beta}{\cos^2 \beta \sin^2 \beta} - 2 \\ &= \frac{1}{\cos^2 \beta \sin^2 \beta} - 2 = \sec^2 \beta \csc^2 \beta - 2 \end{aligned}$$

[3]

$$\sin^4 A + \cos^2 A = \cos^4 A + \sin^2 A$$

$$\begin{aligned} \hookrightarrow &= (1 - \cos^2 A)^2 + \cos^2 A \\ &= 1 - 2\cos^2 A + \cos^4 A + \cos^2 A \\ &= 1 - \cos^2 A + \cos^4 A \\ &= \sin^2 A + \cos^4 A \end{aligned}$$

[4]

$$\sec(-t) - \cos(-t) - \csc(-t) + \sin(-t) + \sin t \tan(-t) = \cos t \cot t$$

$$\begin{aligned} \hookrightarrow &= \sec t - \cos t + \csc t - \sin t - \sin t \tan t \\ &= \frac{1}{\cos t} - \cos t + \frac{1}{\sin t} - \sin t - \sin t \frac{\sin t}{\cos t} \\ &= \frac{1 - \cos^2 t}{\cos t} + \frac{1 - \sin^2 t}{\sin t} - \frac{\sin^2 t}{\cos t} \\ &= \frac{\sin^2 t}{\cos t} + \frac{\cos^2 t}{\sin t} - \frac{\sin^2 t}{\cos t} = \cos t \frac{\cos t}{\sin t} = \cos t \cot t \end{aligned}$$

[5] $(\tan \lambda - \sec \lambda \csc \lambda)(\cot \lambda - \sec \lambda \csc \lambda) = 1$

$$\begin{aligned} L &= \left(\frac{\sin t}{\cos t} - \frac{1}{\cos t \sin t} \right) \left(\frac{\cos t}{\sin t} - \frac{1}{\cos t \sin t} \right) \\ &= \frac{\sin^2 t - 1}{\cos t \sin t} \cdot \frac{\cos^2 t - 1}{\cos t \sin t} \\ &= \frac{-\cos^2 t}{\cos t \sin t} \cdot \frac{-\sin^2 t}{\cos t \sin t} \\ &= 1 \end{aligned}$$

[6] $\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} = 1 - \tan^2 \theta$

$$\begin{aligned} L &= \frac{(\cancel{\cos^2 \theta} + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta} \\ &= 1 - \tan^2 \theta \end{aligned}$$

[7] $\frac{\sin B + \tan B}{1 + \cos B} = \tan B$

$$\begin{aligned} L &= \frac{\sin B + \frac{\sin B}{\cos B}}{1 + \cos B} \\ &= \frac{\sin B \cos B + \sin B}{\cos B} \cdot \frac{1}{1 + \cos B} \\ &= \frac{\sin B (\cos B + 1)}{\cos B} \cdot \frac{1}{1 + \cos B} \\ &= \tan B \end{aligned}$$

[8]

$$\frac{\cot y - \tan y}{\cos y + \sin y} = \frac{\cos y - \sin y}{\cos y \sin y}$$

$$\rightarrow = \frac{\frac{\cos y}{\sin y} - \frac{\sin y}{\cos y}}{\cos y + \sin y}$$

$$= \frac{\cos^2 y - \sin^2 y}{\sin y \cos y} \cdot \frac{1}{\cos y + \sin y}$$

$$= \frac{(\cancel{\cos y + \sin y})(\cos y - \sin y)}{\sin y \cos y (\cancel{\cos y + \sin y})} = \frac{\cos y - \sin y}{\sin y \cos y}$$

[9]

$$\frac{1 - \sin \alpha}{\cos \alpha} = \frac{1}{\sec \alpha + \tan \alpha}$$

$$\rightarrow = \frac{1}{\sec \alpha + \tan \alpha} \cdot \frac{\sec \alpha - \tan \alpha}{\sec \alpha - \tan \alpha}$$

$$= \frac{\sec \alpha - \tan \alpha}{\sec^2 \alpha - \tan^2 \alpha}$$

$$= \frac{1}{\cos \alpha} - \frac{\sin \alpha}{\cos \alpha}$$

$$= \frac{1 - \sin \alpha}{\cos \alpha}$$

[10]

$$\frac{(\sec C - \tan C)^2 + 1}{\sec C \csc C - \tan C \csc C} = 2 \tan C$$

$$\rightarrow = \frac{\sec^2 C - 2 \sec C \tan C + \tan^2 C + 1}{\csc C (\sec C - \tan C)}$$

$$= \frac{\sec^2 C - 2 \sec C \tan C + \sec^2 C}{\csc C (\sec C - \tan C)}$$

$$= \frac{2 \sec^2 C - 2 \sec C \tan C}{\csc C (\sec C - \tan C)}$$

$$= \frac{2 \sec C (\cancel{\sec C - \tan C})}{\csc C (\cancel{\sec C - \tan C})}$$

$$= \frac{2}{\cos C} \cdot \frac{\sin C}{1} = 2 \tan C$$